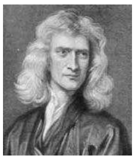
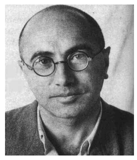



Formation Sciences de la Matière
Cours : Physique Master 2 (ENS)

Cosmologie et Systèmes Gravitationnels

TD 6




Thomas Buchert, CRAL, ENS de Lyon thomas.buchert@ens–lyon.fr
Pierre Mourier, CRAL, ENS de Lyon pierre.mourier@ens–lyon.fr

6.1. Evolution of integrated observables

Show that, for any observable (a tensor field $\mathbf{Q}(\mathbf{x},t)$), we have as a result of restmass conservation on the domain $\mathcal{D}_t$:

$$\frac{d}{dt}\int_{\mathcal{D}_t} \varrho(\mathbf{x},t)\,\mathbf{Q}(\mathbf{x},t)\;d^3x \;=\; \int_{\mathcal{D}_t} \varrho(\mathbf{x},t)\frac{d}{dt}\mathbf{Q}(\mathbf{x},t)\;d^3x \quad . \qquad (6.1)$$

6.2. Commutation rule

Show that, for any tensor field $\mathbf{A}(\mathbf{x},t)$, the following *commutation rule* holds :

$$\frac{d}{dt}\langle\mathbf{A}\rangle_{\mathcal{D}_t} - \langle\frac{d}{dt}\mathbf{A}\rangle_{\mathcal{D}_t} = \langle\mathbf{A}\theta\rangle_{\mathcal{D}_t} - \langle\mathbf{A}\rangle_{\mathcal{D}_t}\langle\theta\rangle_{\mathcal{D}_t} \quad . \qquad (6.2)$$

6.3. Newton's iron spheres

Show that, for spherically symmetric mean velocity field and vanishing effective velocity dispersion force, $\langle\psi_{i,i}\rangle_{\mathcal{B}_t} = 0$, the *backreaction* term $\mathcal{Q}_{\mathcal{B}_t}$ vanishes on a spherical domain $\mathcal{B}_t$ with radius $r = r(\mathcal{R},t)$.

Hint : $(\boldsymbol{\nabla}\cdot\mathbf{v}) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 S(r)\right)$, et $(\mathbf{v}\cdot\boldsymbol{\nabla}) = S(r)\frac{d}{dr}$, for a velocity potential S.

6.4. Dark Energy from inhomogeneities ?

Employ the *Hamilton constraint* of general relativity, given in TD 2.3, and construct its spatial average using the definition

$$\langle \Psi \rangle_{\mathcal{D}} := \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \Psi J d^3 X \quad , \tag{6.3}$$

for a scalar field Ψ. Note that, In general relativity, the function J is given by $J := \sqrt{\det(g_{ij})}$ with the Riemannian metric coefficients g_{ij}, but the identity $\dot{J} = J\Theta$ still holds.

Consider now a practically empty ($\langle \varrho \rangle_{\mathcal{D}} \cong 0$) and expanding ($\langle \theta \rangle_{\mathcal{D}} > 0$) universe model.

Argue whether a universe model can exist, where the (positive) cosmological constant is replaced by invariants of the expansion tensor, while expanding almost isotropically on average.

What would you say about the averaged intrinsic curvature of this universe model ?

What could happen in the case of a contracting domain $\mathcal{D}$ ($\langle \Theta \rangle_{\mathcal{D}} < 0$) that is not isotropic ?

Hint : Employ a kinematical decomposition of the expansion tensor, and introduce the backreaction term $\mathcal{Q}_{\mathcal{D}}$ for vanishing vorticity and dispersion.